Linear - Quiz 2

11:34 12:28
11:36 12:37

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give 20 minutes.

Name: Key

1. (4 points) Let u= $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and v= $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$. Show that $\begin{bmatrix} a \\ b \end{bmatrix}$ is in Span{u, v} for all a and b.

$\left[\begin{array}{cc|c} 3 & 3 & a \\ -2 & 2 & b \end{array}\right]$ is aug. matrix ✓

$\frac{1}{3}R_1 \to R_1 \left[\begin{array}{cc|c} 1 & 1 & a/3 \\ -2 & 2 & b \end{array}\right]$ ✓

$2R_1+R_2 \to R_2 \left[\begin{array}{cc|c} 1 & 1 & a/3 \\ 0 & 4 & \frac{2}{3}a+b \end{array}\right]$ $\frac{1}{2}$✓

This is in echelon form, since the bottom row, 2nd column is $\neq 0$, this will be consistent. So for any a, b, the matrix will have a solution $\Leftrightarrow \begin{bmatrix} a \\ b \end{bmatrix}$ is in Span{u,v}

✓ 1.5

2. (2 points) Construct a 4×4 matrix, not in echelon form, whose columns span $\mathbb{R}^4$. Show that the matrix you construct has the desired property. Hint: Keep it simple, but not in echelon form.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$-R_1+R_4 \to R_4$ gives $\begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{bmatrix}$

which clearly span $\mathbb{R}^4$ because it has a pivot in every column.

Many Answers.

3. (4 points) Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$. Does $\{v_1, v_2, v_3\}$ span $\mathbb{R}^4$? Why or why not?

No. ✓

Make an augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & b_1 \\ 0 & -1 & 0 & b_2 \\ -1 & 0 & 0 & b_3 \\ 0 & 1 & -1 & b_4 \end{array}\right]$$

✓

$R_1 + R_3 \to R_3$

$R_2 + R_4 \to R_4$ $$\left[\begin{array}{ccc|c} 1 & 0 & 1 & b_1 \\ 0 & -1 & 0 & b_2 \\ 0 & 0 & 1 & b_1+b_3 \\ 0 & 0 & -1 & b_2+b_4 \end{array}\right]$$

✓

$R_3 + R_4 \to R_4$ $$\left[\begin{array}{ccc|c} 1 & 0 & 1 & b_1 \\ 0 & -1 & 0 & b_2 \\ 0 & 0 & 1 & b_1+b_3 \\ 0 & 0 & 0 & b_1+b_2+b_3+b_4 \end{array}\right]$$

Any choice of $b_1, \dots, b_4$ such that $b_1+b_2+b_3+b_4 \neq 0$ ✓ will make the matrix inconsistent $\Longrightarrow \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$ will <u>not</u> be in span $\{v_1, v_2, v_3\}$. For example, $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \notin \text{span}\{v_1, v_2, v_3\}$